Baryon diffusion in heavy-ion collisions

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RIKEN BNL Center Workshop

"Theory and Modeling for the Beam Energy Scan: from Exploration to Discovery"

27th February 2015, BNL, NY, USA

Overview

AM, Phys. Rev. C 86, 014908 (2012)

B. Schenke and AM, in preparation

To collaborate with G. Denicol, C. Shen, S. Jeon and C. Gale (McGill)

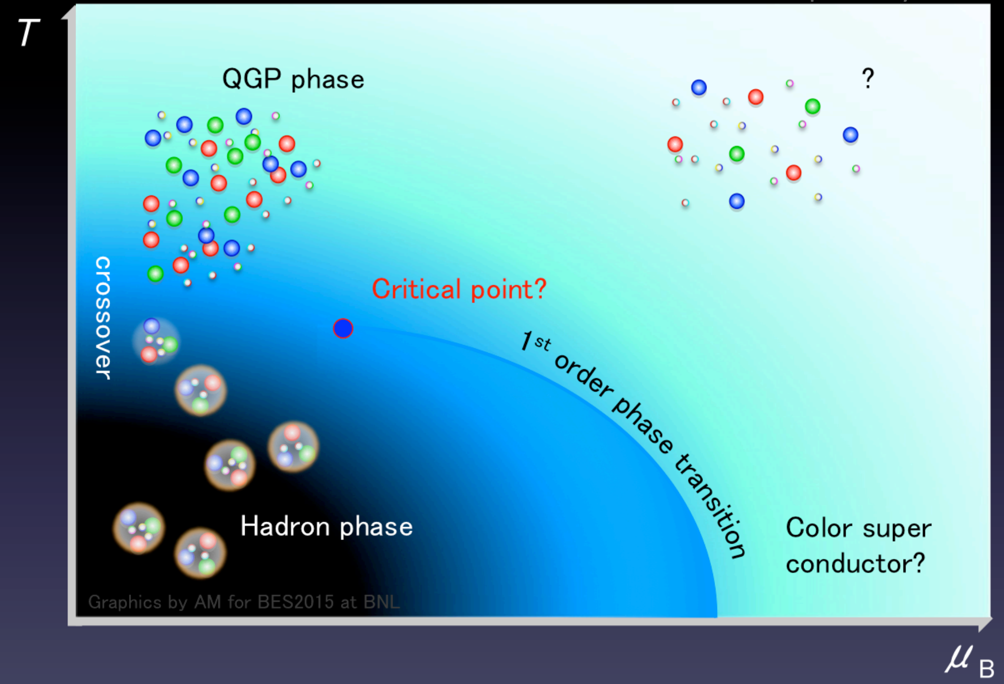
Introduction

- Beam energy scans: exploration of QCD phase diagram in heavy-ion collisions

Big goals:

- Explicate the QGP properties at finite μ_B
- Search for a QCD critical point

Graphics by AM



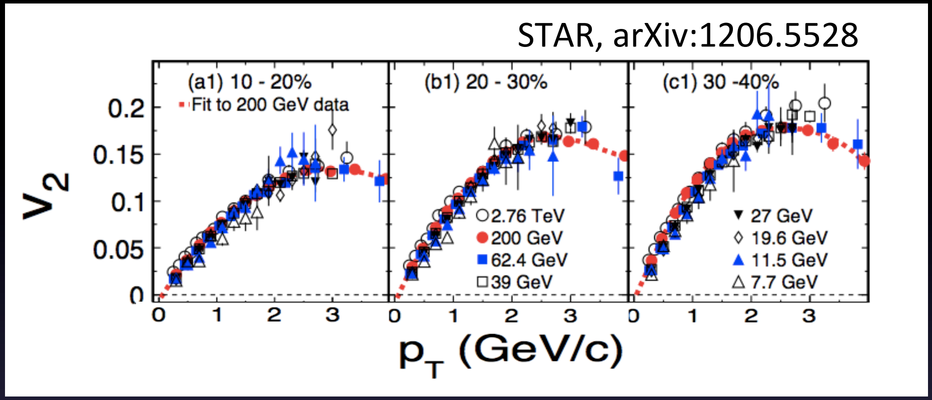
- Hydrodynamic approaches

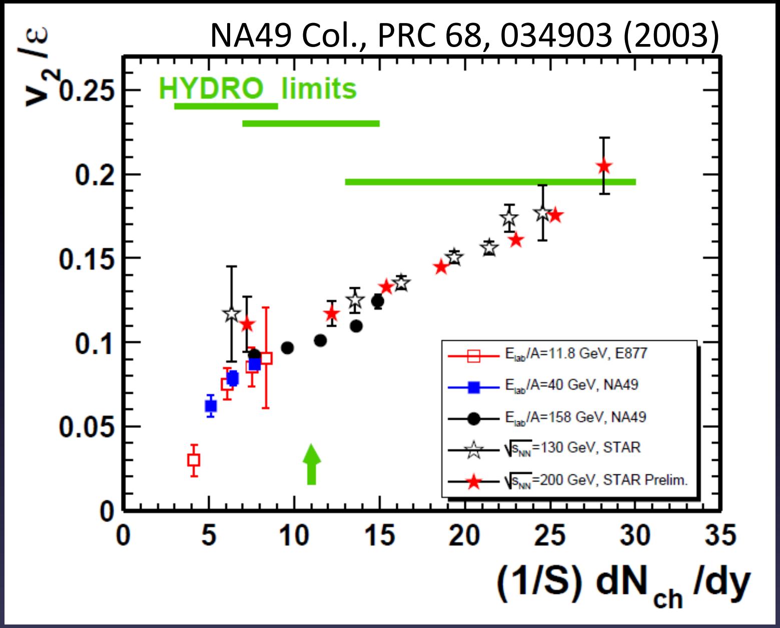
The QGP at high energy is quantified as a relativistic fluid (2000)

We consider dissipative hydrodynamics at finite densities

Introduction

- Is hydrodynamics applicable?





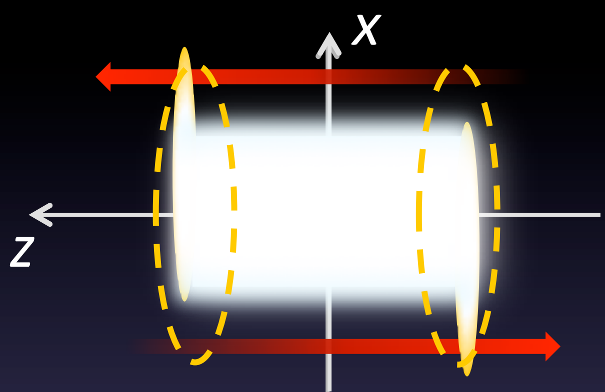
- Differential v_2 is large
- Integrated v_2 stays positive above $\sqrt{s_{NN}}$ ~ 3 GeV but is small

We will see, with off-equilibrium corrections, finite-density effects, state-of-art initial conditions and EoS

Introduction

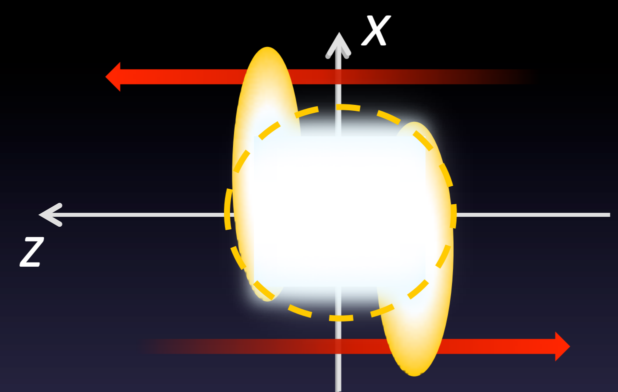
- Schematic pictures of collision geometries

At high-energies



Net baryon at forward rapidity

At low-energies



Net baryon at mid-rapidity

Finite-density hydro is relevant in

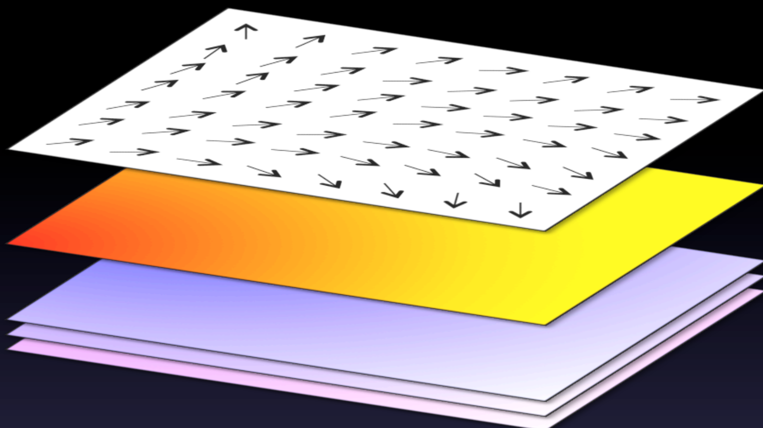
- Particle identification analyses (p/$\bar{p}$ ratio, etc.)
- Quantification of transport properties
- Bulk evolution for low energy collisions?

2. Dissipative hydrodynamics

Reference: AM, Phys. Rev. C 86, 014908 (2012)

Relativistic hydrodynamics

- Local thermalization; macroscopic variables are defined as fields



Flow $u^\mu(x)$ $\quad u^\mu u_\mu = 1$

Temperature $T(x)$

Chemical potentials $\mu_J(x)$

⇨ *Gradient in the fields*: thermodynamic force

Response to the gradients: transport coefficients (= 0 if ideal hydro)

- Energy-momentum tensor & conserved current are

$$T^{\mu\nu} = (e_0 + \delta e)u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + 2W^{(\mu}u^{\nu)} + \pi^{\mu\nu}$$
$$N_J^\mu = (n_{J0} + \delta n_J)u^\mu + V_J^\mu$$

when decomposed with u^μ ; $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Thermodynamic quantities

- In local rest frame $u^\mu = (1,0,0,0)$

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$$

$$= \begin{pmatrix} e_0 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{pmatrix} + \begin{pmatrix} \delta e & W^x & W^y & W^z \\ W^x & \Pi + \pi^{xx} & \pi^{xy} & \pi^{xz} \\ W^y & \pi^{yx} & \Pi + \pi^{yy} & \pi^{yz} \\ W^z & \pi^{zx} & \pi^{yz} & \Pi + \pi^{zz} \end{pmatrix}$$

$$N_J^\mu = N_{J0}^\mu + \delta N_J^\mu \quad (J = 1,2,\ldots,N)$$

$$= \begin{pmatrix} n_{J0} \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \delta n_J \\ V_J^x \\ V_J^y \\ V_J^z \end{pmatrix}$$

2+*N* equilibrium quantities

Energy density: e_0
Hydrostatic pressure: P_0
J-th charge density: n_{J0}

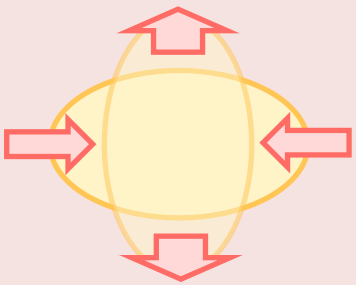
10+4*N* dissipative currents

Energy density deviation: δe
Bulk pressure: Π
Energy current: W^μ
Shear stress tensor: $\pi^{\mu\nu}$
J-th charge density dev.: δn_J
J-th charge current: V_J^μ

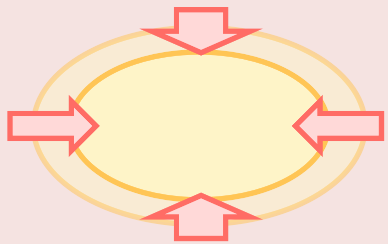
Viscosity and diffusion

- Meaning of “dissipation” in fluids

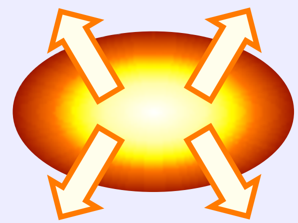
Off-equilibrium processes at linear order



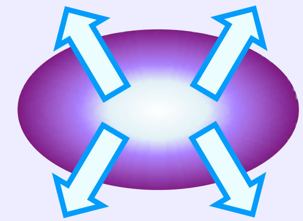
Shear viscosity
= response to
deformation



Bulk viscosity
= response to
expansion



Energy dissipation
= response to
thermal gradient



Charge diffusion
= response to
chemical gradients

viscosity

dissipation/diffusion

- Cross terms among thermodynamic forces are present (discussed later)
- 2nd order corrections are required for hydrodynamic stability and causality

W. Israel, J. M. Stewart, Annals Phys 118, 341 (1979)
W.A. Hiscock, L. Lindblom, Phys. Rev. D 31, 725 (1985)

Dissipative hydrodynamics

- Relativistic hydrodynamic equations

Conservation laws $\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu N^\mu_B = 0$ +

$D = u^\mu \partial_\mu$
$\nabla^\mu = \partial^\mu - u^\mu D$

The law of increasing entropy -> Constitutive equations

$$\Pi = -\zeta \nabla_\mu u^\mu - \zeta_{\Pi\delta e} D\frac{1}{T} + \zeta_{\Pi\delta n_B} D\frac{\mu_B}{T} - \tau_\Pi D\Pi + \chi^a_{\Pi\Pi}\Pi D\frac{\mu_B}{T} + \chi^b_{\Pi\Pi}\Pi D\frac{1}{T} + \chi^c_{\Pi\Pi}\Pi\nabla_\mu u^\mu$$
$$+ \chi^a_{\Pi V} V_\mu \nabla^\mu \frac{\mu_K}{T} + \chi^b_{\Pi V} V_\mu \nabla^\mu \frac{1}{T} + \chi^c_{\Pi V} V_\mu D u^\mu + \chi^d_{\Pi V}\nabla^\mu V_\mu + \chi_{\Pi\pi}\pi_{\mu\nu}\nabla^{\langle\mu}u^{\nu\rangle}$$

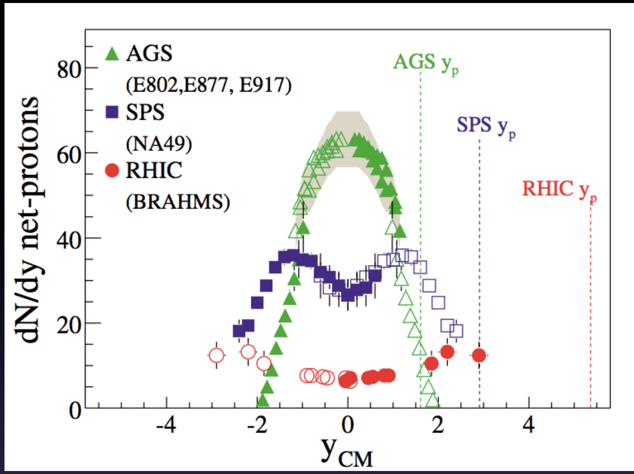
$$V^\mu = \kappa_V \nabla^\mu \frac{\mu_B}{T} - \kappa_{VW}\left(\frac{1}{T}Du^\mu + \nabla^\mu \frac{1}{T}\right) - \tau_V \Delta^{\mu\nu} D V_\nu + \chi^a_{VV} V^\mu_K D\frac{\mu_B}{T} + \chi^b_{VV} V^\mu D\frac{1}{T}$$
$$+ \chi^c_{V_J V} V^\mu \nabla_\nu u^\nu + \chi^d_{VV} V^\nu_K \nabla_\nu u^\mu + \chi^e_{VV} V^\nu \nabla^\mu u_\nu + \chi^a_{V\pi}\pi^{\mu\nu}\nabla_\nu \frac{\mu_B}{T} + \chi^b_{V\pi}\pi^{\mu\nu}\nabla_\nu \frac{1}{T}$$
$$+ \chi^c_{V\pi}\pi^{\mu\nu} D u_\nu + \chi^d_{V\pi}\Delta^{\mu\nu}\nabla^\rho \pi_{\nu\rho} + \chi^a_{V\Pi}\Pi\nabla^\mu \frac{\mu_B}{T} + \chi^b_{V\Pi}\Pi\nabla^\mu \frac{1}{T} + \chi^c_{V\Pi}\Pi D u^\mu + \chi^d_{V\Pi}\nabla^\mu \Pi$$

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle} - \tau_\pi D\pi^{\langle\mu\nu\rangle} + \chi_{\pi\Pi}\Pi\nabla^{\langle\mu}u^{\nu\rangle} + \chi^a_{\pi\pi}\pi^{\mu\nu} D\frac{\mu_B}{T} + \chi^b_{\pi\pi}\pi^{\mu\nu} D\frac{1}{T} + \chi^c_{\pi\pi}\pi^{\mu\nu}\nabla_\rho u^\rho$$
$$+ \chi^d_{\pi\pi}\pi^{\rho\langle\mu}\nabla_\rho u^{\nu\rangle} + \chi^{aJ}_{\pi V} V^{\langle\mu}\nabla^{\nu\rangle}\frac{\mu_B}{T} + \chi^b_{\pi V} V^{\langle\mu}\nabla^{\nu\rangle}\frac{1}{T} + \chi^c_{\pi V} V^{\langle\mu} D u^{\nu\rangle} + \chi^d_{\pi V}\nabla^{\langle\mu}V^{\nu\rangle}$$

Numerical analyses

- Baryon stopping

Plot: BRAHMS, PRL 93, 102301 (2004)



Baryon stopping can quantify kinetic energy available for QGP production

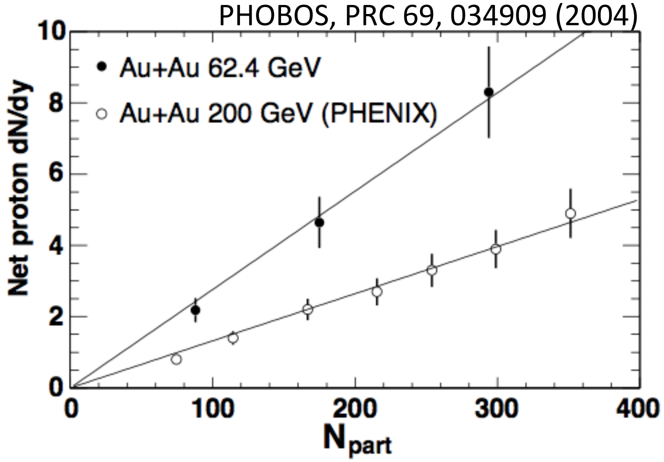
mean rapidity loss $\langle \delta y \rangle$
= rapidity of projectile nuclei y_b
– mean rapidity of net baryon $\langle y \rangle$

- What we do:

▶ Estimate dissipative hydro evolution of net baryon rapidity distribution with viscosities and baryon diffusion

(1+1)-D expansion is considered because dependence on transverse geometry is small

PHOBOS, PRC 69, 034909 (2004)



Simulation Setup

- Equation of state: Lattice QCD with Taylor expansion

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_B^{(2)}(T,0)}{2}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4$$

$P(T,0)$: Equation of state at vanishing μ_B — S. Borsanyi *et al.*, JHEP 1011, 077

$\chi_B^{(2)}(T,0)$: 2nd order baryon fluctuation — S. Borsanyi *et al.*, JHEP 1201, 138

- Transport coefficients: AdS/CFT + phenomenology

Shear viscosity: $\eta = s/4\pi$ — P. Kovtun *et al.*, PRL 94, 111601

Bulk viscosity: $\zeta = 5(\frac{1}{3} - c_s^2)\eta$ — A. Hosoya *et al.*, AP 154, 229

Baryon dissipation: $\kappa_V = \frac{c_V}{2\pi}\left(\frac{\partial \mu_B}{\partial n_B}\right)_T^{-1}$ — M. Natsuume and T. Okamura, PRD 77, 066014

- Initial conditions: Color glass theory

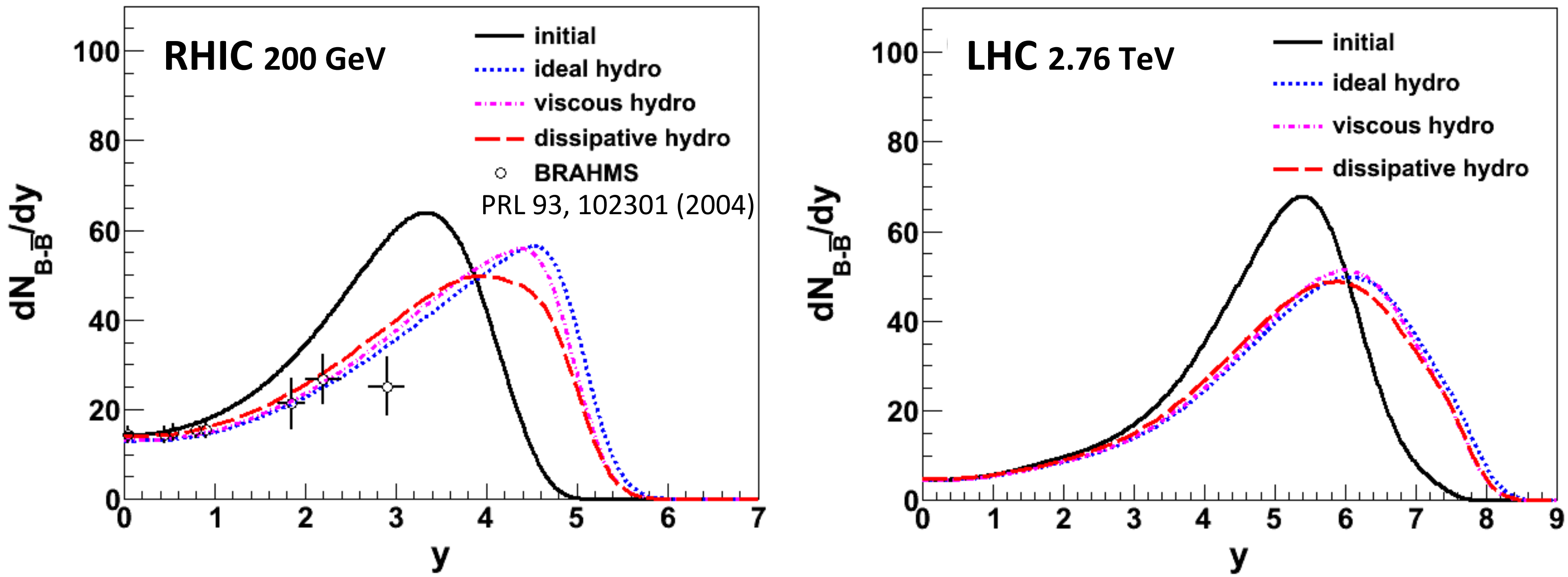
Energy density: MC-KLN

Net baryon density: Valence quark dist.

H. J. Drescher and Y. Nara, PRC 75, 034905; 76, 041903
Y. Mehtar-Tani and G. Wolschin, PRL 102, 182301; PRC 80, 054905

Results

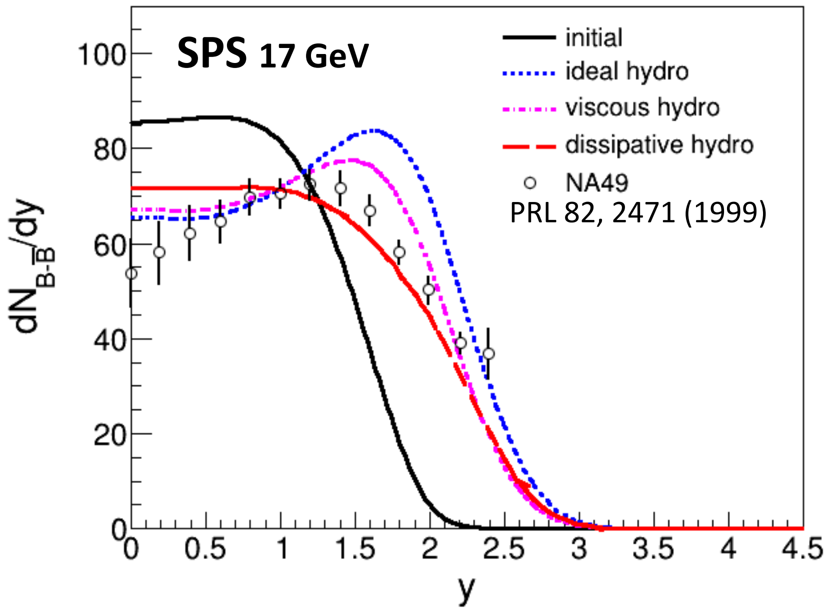
- Net baryon rapidity distribution at RHIC and LHC

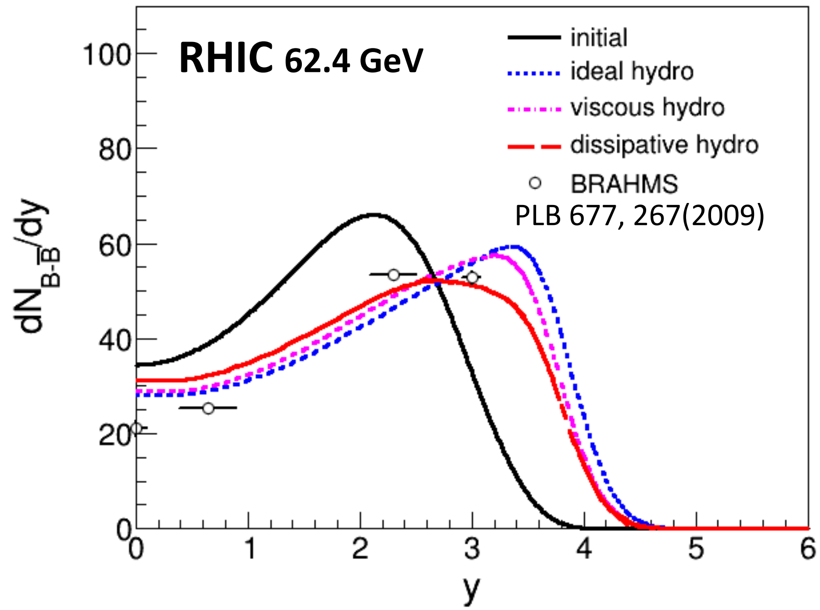


- Net baryon is carried to forward rapidity by convection
- Viscosities slow the longitudinal expansion
- Net baryon diffuses into mid-rapidity

Results

- Net baryon rapidity distribution at SPS and RHIC





- Results can be comparable to data (not fine-tuned yet)
- Dissipative effect could be larger for lower energies

Note: CGC-based initial conditions (not best suitable at low energies)

Results

- Mean rapidity loss at RHIC

Mean rapidity loss $\langle \delta y \rangle = y_p - \langle y \rangle$

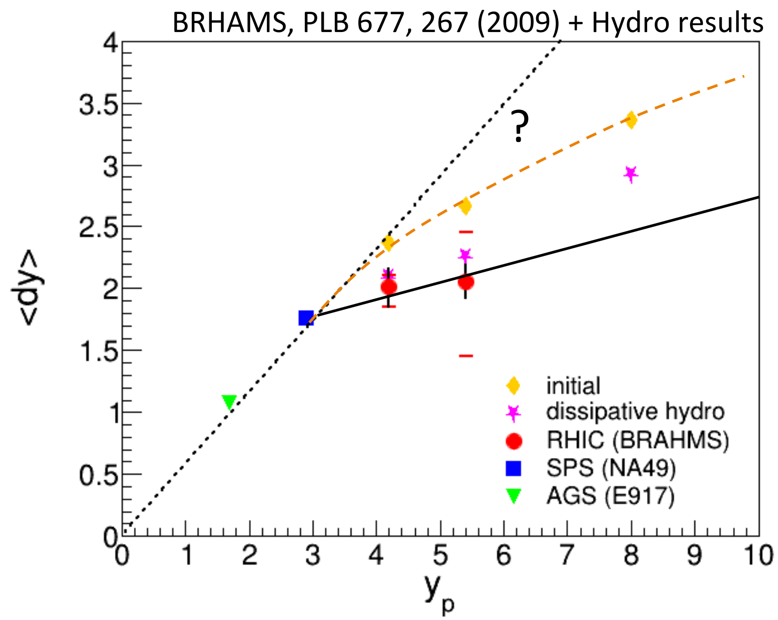
$$\langle y \rangle = \int_0^{y_p} y \frac{dN_{B-\bar{B}}(y)}{dy} dy \Big/ \int_0^{y_p} \frac{dN_{B-\bar{B}}(y)}{dy} dy$$

Initial loss (200GeV): $\langle \delta y \rangle = 2.67$

Ideal hydro: $\langle \delta y \rangle = 2.09$

Viscous hydro: $\langle \delta y \rangle = 2.16$

Dissipative hydro: $\langle \delta y \rangle = 2.26$



- The collision becomes effectively more transparent by hydrodynamic evolution

⇒ More kinetic energy is available for QGP production

Cross-coupling effects (1)

- Linear response theory and cross terms

Bulk pressure (w/o charges)

$$\Pi = -\zeta_{\Pi\Pi}\frac{1}{T}\nabla_\mu u^\mu - \zeta_{\Pi\delta e}D\frac{1}{T} = -\underbrace{\left(\frac{\zeta_{\Pi\Pi}}{T} + \frac{\zeta_{\Pi\delta e}}{T}c_s^2\right)}_{\text{bulk viscosity } \zeta}\nabla_\mu u^\mu$$

Response to expansion

Response to cooling

- Response to expansion itself can be as large as shear viscosity
- Cancelled by the cross term except for crossover where $c_s^2 \sim 0$

⇨ A reason for general smallness of bulk viscosity

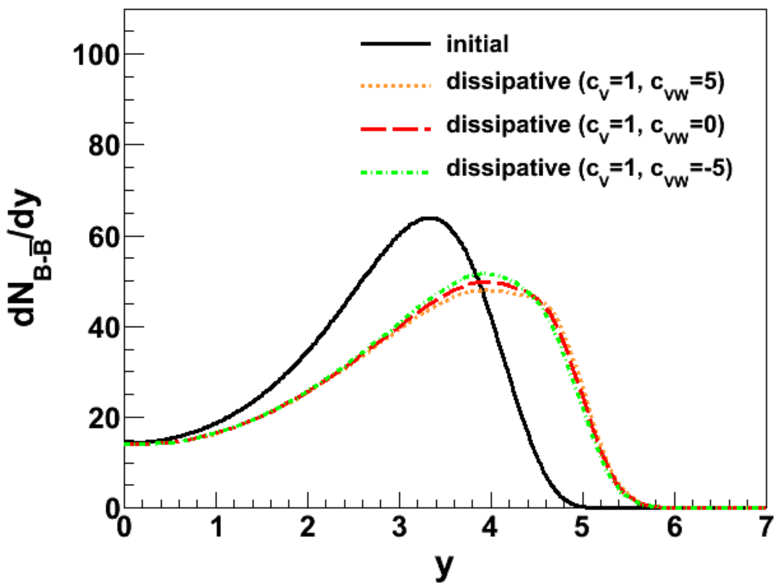
Baryon dissipation current

$$V^\mu = \kappa_V \nabla^\mu \frac{\mu_B}{T} - \kappa_{VW}\left(\nabla^\mu \frac{1}{T} + \frac{1}{T}Du^\mu\right)$$

- Baryon dissipation can be induced by thermal gradient + acceleration

Results

- Thermo-diffusion effect (a.k.a. Soret effect)



- Baryon dissipation can be induced by thermal gradients (and acceleration)

$$V^\mu = \kappa_V \nabla^\mu \frac{\mu_B}{T} - \kappa_{VW} \left(\nabla^\mu \frac{1}{T} + \frac{1}{T} D u^\mu \right)$$

at the linear order

- Cross coefficients can be negative if the coefficient matrix is positive definite

▪ The effect of cross coupling is likely to be small in high-energy collisions

because of the matter-antimatter symmetry

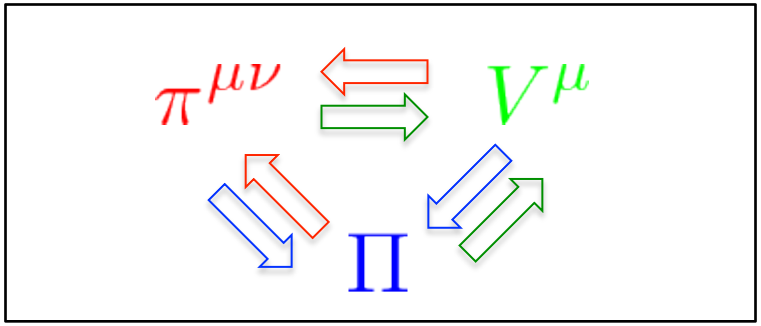
$V^\mu(\mu_B) = -V^\mu(-\mu_B)$ which leads to $\kappa_{VW}(\mu_B = 0) = 0$

Cross-coupling effects (2)

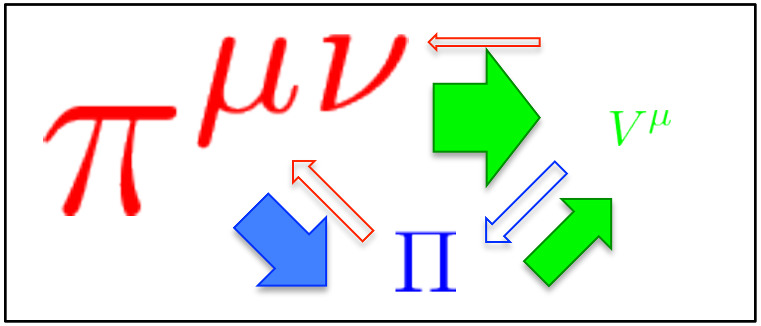
- Mixing of the currents at the 2nd order

System dependence

Hydrodynamic theory considers:
$\pi^{\mu\nu} \sim \Pi \sim V^{\mu}$



In high-energy nuclear collisions:
$\pi^{\mu\nu} > \Pi > V^{\mu}$



- Bulk-shear coupling term in bulk pressure
  Baryon-shear and baryon-bulk coupling terms in baryon dissipation
  have more impact than other 2nd order terms (numerically confirmed)
- Applicability of the expansion is dependent on the 2nd order transport coefficients

Summary so far

- Dissipative hydrodynamic model is developed and simulated in (1+1)D at finite baryon density
  - Net baryon distribution is widened in hydrodynamic evolution
    - ⇨ Transparency of the collision is effectively enhanced
    - ⇨ More kinetic energy may be available at QGP (and jet) production in early stages
  - The results can be sensitive to baryon diffusion coefficient
    - ⇨ Ambiguities remain in initial condition, but the distribution has important information
  - Hydrodynamic results for baryon stopping are comparable to the experimental data at lower energies

3. Towards full analyses of BES

B. Schenke and AM

To collaborate with G. Denicol, C. Shen, S. Jeon and C. Gale

Initial conditions

- 3D Monte-Carlo Glauber model

Net baryon distribution

Valence quark PDF for the rapidity distribution before collisions

⇨ A collision modifies the distribution via the kernel

S. Jeon and J. Kapusta, PRC 56, 468

$$Q(y - y_P, y_P - y_T, y - y_P) = \lambda \frac{\cosh(y - y_P)}{\sinh(y_P - y_T)} + (1 - \lambda)\delta(y - y_P)$$

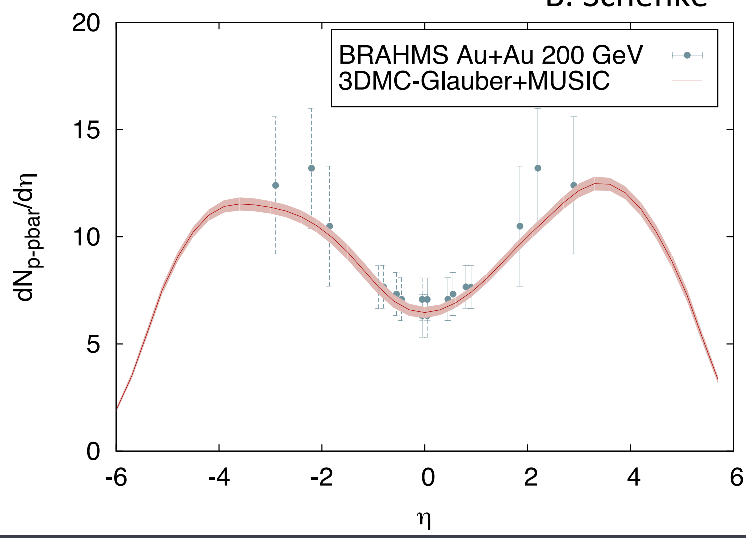
⇨ Keep sampling for all the parton-parton collisions

Entropy distribution

Entropy is deposited between the last collision pairs

A simple and straight-forward extension of 2D MC Glauber model

B. Schenke



Equation of state

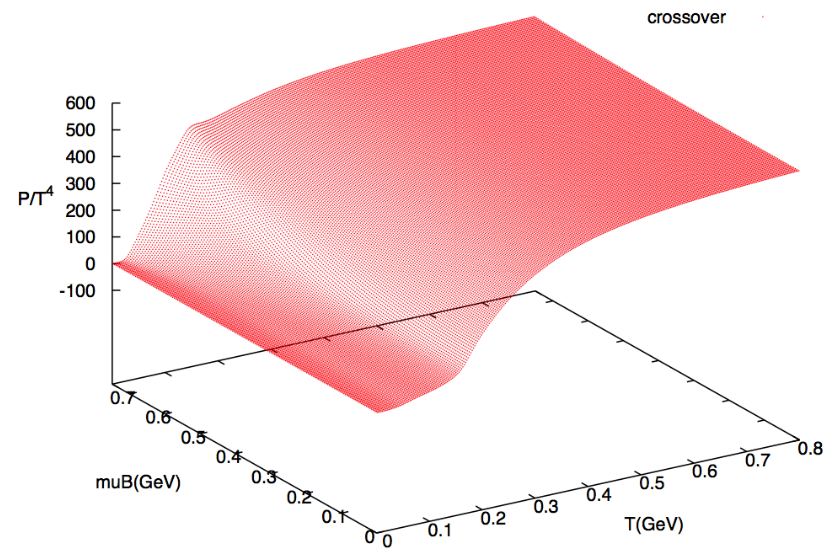
- Lattice QCD (Taylor expansion) + Hadron resonance gas

$$\frac{P}{T^4} = \frac{1}{2}\left[1 - \tanh\frac{T - T_c(\mu_B)}{\Delta T_c}\right]\frac{P_{\mathrm{HRS}}(T)}{T^4} + \frac{1}{2}\left[1 + \tanh\frac{T - T_c(\mu_B)}{\Delta T_c}\right]\frac{P_{\mathrm{lat}}(T_s)}{T_s^4}$$

where

$$T_c = 0.166 - c(0.139\mu_B^2 + 0.053\mu_B^4)$$

$$T_s = T + c[T_c(0) - T_c(\mu_B)]$$



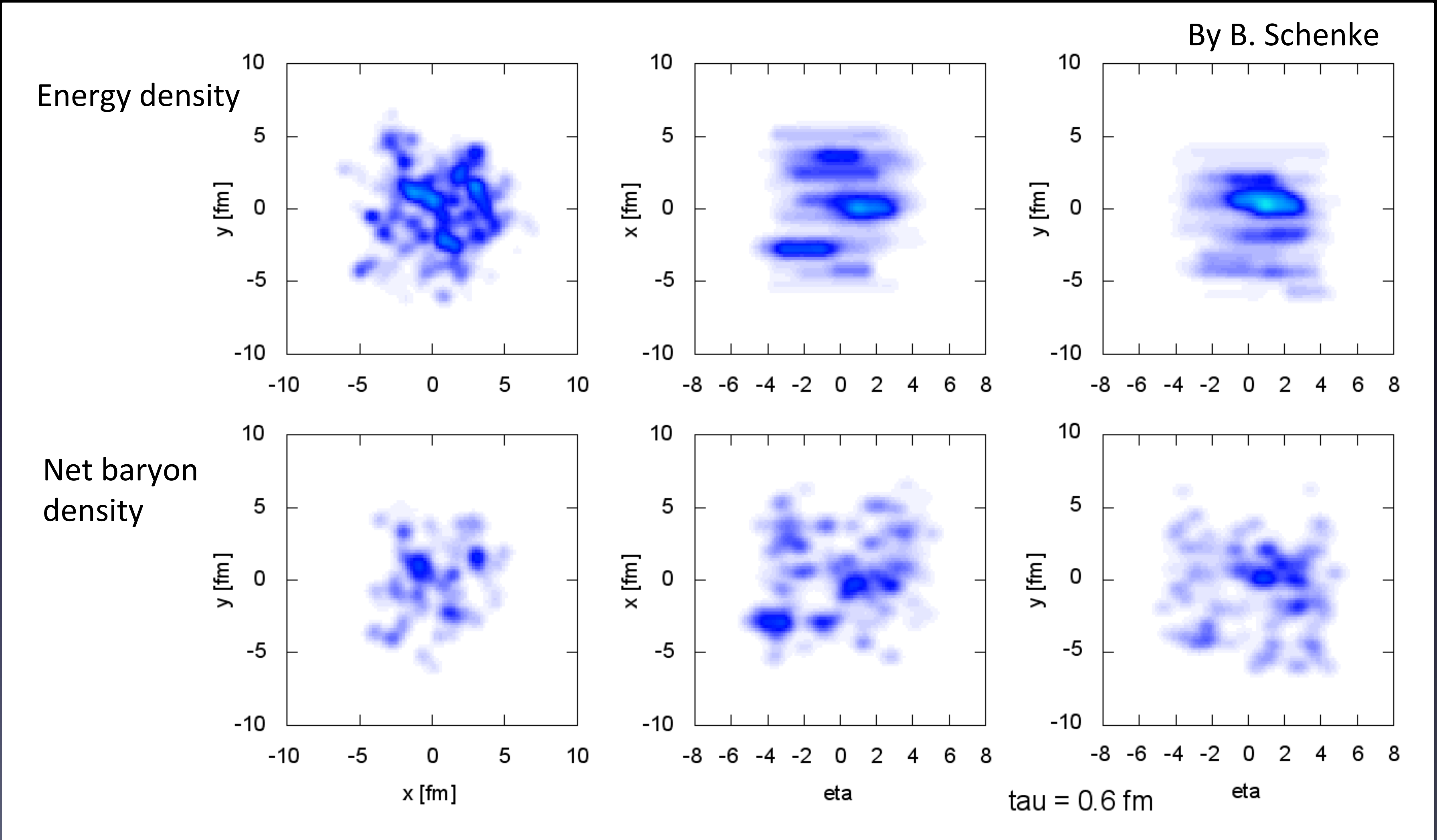
EoS of kinetic theory must match EoS for hydrodynamics at freeze-out (or energy-momentum/net baryon does not conserve)

u^μ, T, μ_B

Particle spectrum $\quad E_i \frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_\Sigma p_i^\mu d\sigma_\mu f_i \quad$ Hydrodynamics

Hydrodynamic evolution

- 3+1 D event-by-event analyses (work in progress)



4. Summary and outlook

Summary and outlook

- (3+1)-D event-by-event hydrodynamic model at finite baryon density in preparation
  - Initial condition: 3D Monte-Carlo Glauber model
  - Equation of state: Lattice QCD with Taylor expansion method
    + Hadron resonance gas
  - Viscosity: shear viscosity + bulk viscosity
  - Baryon diffusion: *see next talk by Chun*

- Thank you for listening!